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The phases of 2D NCOS

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Abstract: We study the phases of the (1+1)-dimensional Non-Commutative Open String theory on a circle. We find that the length scale of non-commutativity increases at strong coupling, the coupling in turn being dressed by a power of D-string charge. The system is stringy at around this length scale, with dynamics involving an interplay between the open and wrapped closed strings sectors. Above this energy scale and at strong coupling, and below it at weak coupling, the system acquires a less stringy character. The near horizon geometry of the configuration exhibits several intriguing features, such as a flip in the dilaton field and the curvature scale, reflecting UV-IR mixing in non-commutative dynamics. Two special points in the parameter measuring the size of the circle are also identified.

Keywords: Superstrings and Heterotic Strings, D-branes, M(atrix) Theories, AdS/CFT Correspondance.

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1. Introduction and discussion

It has long been suspected that non-commutativity of space-time coordinates is a key ingredient in the proper formulation of Planck scale physics [1, 2]. In the context of string theory, this phenomenon has risen as a characteristic feature of strong coupling dynamics. A common theme in exploring new string physics has been to arrange for systems where an unknown regime of the theory can be studied through a dual formulation. The Holographic duality [3] for example associates a low energy sector of certain open string theories with supergravity, leading to a better understanding of Super Yang-Mills (SYM) theories at strong coupling. In this setup however, either too much of the interesting string dynamics is scaled away, so that one essentially learns about field theories; or the stringy remnant in the decoupled regime is not very well understood, as in the scenario of little string theories. It would naturally be desirable to find settings where more of stringy dynamics, such as non-commutativity of spacetime coordinates, survives the decoupling limit. Indeed, in the work of [4], it has been shown that this may be achieved by adding another charge to certain D-brane systems. One is then lead to a spectrum of new string/field theories on non-commutative spaces [4]–[12]. And through the Holographic duality, even the strong coupling regimes of these theories can now be explored [13, 14].

In this context, a particularly interesting setup is obtained by adding fundamental string charge to a system of D-strings. The resulting bound state of strings and D-strings can be studied through a $(1+1)$-dimensional Non-Commutative Open String theory (NCOS) [5, 15]. This system appears to be the simplest one that explores non-commutative dynamics; its role in understanding this phenomenon may be as fundamental as that of the role of D0-branes in understanding Dp-brane dynamics [2]. The theory, unlike little string theories and OM theory, has a well defined perturbative expansion. It also necessarily involves non-commutativity in the time coordinate; all this within the framework of a self-consistent, Lorentz invariant, kinematically simple and computationally accessible string theory. It is then a good candidate for a framework to put one's intuition with regards to non-commutativity to extreme tests. Finally, the particularities of non-commutativity involving the time variable may yield new insight in understanding information transfer and dynamics near black hole horizons.

In this work, we attempt to understand this theory in a thermodynamic setting, in the hope that such a macroscopic analysis would be a guide to identifying some of the new interesting dynamical consequences of time-space non-commutativity. We expect that critical phenomena in the system would reflect the peculiarities of the underlying microscopic dynamics.

Most of the technical aspects of our analysis will parallel similar ones that have appeared in the literature [16] in the context of commutative Yang-Mills theories. In the next section, we briefly review the theory of interest. In section 1.2, we present the phase diagram of $(1+1)$-dimensional NCOS theory on a circle, with a discussion of the physics underlying the various phases. In section 1.3, we summarize the new results of the paper, and suggest future directions. Section 2 contains all the details of the construction of the phase diagram and may be skipped without much grief to anyone. Finally, figure 4 contains a roadmap used extensively in the text of section 2.

Note added. The article [17] also studies this system, as well as the higher-dimensional cases, and reaches some of the same conclusions.

1.1 Preliminaries

In this section, we define the $(1+1)$-dimensional NCOS theory [5, 15] and set the notation and conventions used in the paper. We start in IIB theory in the presence of a bound state of N fundamental strings and M D-strings, wrapped on a coordinate y of size Σ. The theory is parameterized by the string coupling $g_{\rm str}$ and string scale α'. We choose coordinates such that the metric is given by

$$g_{\mu\nu} = g_{\rm str}\eta_{\mu\nu}\,. \tag{1.1}$$

The NSNS B field then becomes

$$B^2 \equiv \frac{(2\pi\alpha')^2 B_{ty}^2}{g_{\rm str}^2} = \left(1+\left(\frac{M}{Ng_{\rm str}}\right)^2\right)^{-1}\,. \tag{1.2}$$

The dynamics of this bound state can be described by a $(1+1)$-dimensional theory of open strings propagating on a non-commutative space with metric

$$G_{\mu\nu} = g_{\rm str}\left(1-B^2\right)\eta_{\mu\nu}\,. \tag{1.3}$$

The string endpoints carry M Chan-Paton indices and are confined to the non-commutative $t - y$ plane

$$[t, y] = i\theta \tag{1.4}$$

with the parameter θ given by

$$\theta = \frac{B^2}{B_{ty}\left(1 - B^2\right)} . \tag{1.5}$$

The open string coupling G_o becomes

$$G_o^2 = G_s = g_{\rm str}\sqrt{1 - B^2} . \tag{1.6}$$

It was argued in [4, 5, 15] that there exists an energy regime where the dynamics of these open strings decouples from the closed string sector in the bulk. This decoupling limit is obtained by

$$\alpha' \longrightarrow 0 \text{ while keeping } g_{\rm str}\alpha' \text{ fixed} . \tag{1.7}$$

The NSNS B field (or equivalently the electric field on the D strings) then attains its maximal value

$$1 - B^2 \longrightarrow \left(\frac{M}{N g_{\rm str}}\right)^2 \longrightarrow 0 . \tag{1.8}$$

The metric becomes

$$G_{\mu\nu} \longrightarrow g_{\rm str}^{-1}\left(\frac{M}{N}\right)^2 \eta_{\mu\nu} , \tag{1.9}$$

with θ given by

$$\theta \longrightarrow 2\pi g_{\rm str}\alpha'\left(\frac{N}{M}\right)^2 \equiv 2\pi\alpha_e . \tag{1.10}$$

The NCOS string scale $\alpha_e = {l_e}^2$ is the fundamental length scale of this new string theory. While the open string coupling becomes a rational number

$$G_s \longrightarrow \frac{M}{N} . \tag{1.11}$$

This limit is survived by the Virasoro tower of open string excitations since

$$G^{tt}E^2 \sim \frac{N_{\rm osc}}{\alpha'} \Longrightarrow E^2 \sim \frac{N_{\rm osc}}{\alpha_e} ; \tag{1.12}$$

along with closed strings that wrap the cycle y. The box size seen by the NCOS theory is Σ and can be determined by looking at the dispersion relation of a wrapped closed string [15]

$$\begin{aligned} {\alpha'}^2 E^2 &= \left(g_{\rm str}\omega\Sigma\right)^2\left(1 - B^2\right) - 2EB\omega\Sigma g_{\rm str}\alpha' + 2g_{\rm str}\alpha' N_{\rm osc} \\ &\Longrightarrow E = \frac{\omega\Sigma}{2\alpha_e} + \frac{N_{\rm osc}}{\omega\Sigma} , \end{aligned} \tag{1.13}$$

where ω is the winding number. In the S-dual frame, $g_{\rm str} \to 1/g_{\rm str}$ and $g_{\mu\nu} \to g_{\mu\nu}/g_{\rm str} = \eta_{\mu\nu}$. Then the NSNS field scales as $B^2 \to 1/g_{\rm str} \to 0$, while the dual non-commutative parameter scales to zero as well $\theta \to \alpha'/\sqrt{g_{\rm str}} \to 0$. The S-dual theory is decoupled $(1+1)$-dimensional U(N) SYM theory with M units of electric flux [5, 15].

1.2 The phase diagram

The theory of interest is $(1+1)$-dimensional NCOS theory as defined in [5] and summarized in the previous section. Alternatively, we are studying the phase diagram of $(1+1)$-dimensional U(N) SYM theory with electric flux on a compact cycle. The NCOS parameter space consists of the NCOS string scale l_e, the string coupling G_o, the size of the compact cycle Σ, and the integer M counting the number of D-strings. On the SYM side, the parameters are the dimensionful Yang-Mills coupling ${g_Y}^2$, the size of the circle Σ, the rank of the gauge group N, and the M units of electric flux. This entire setup can also be embedded in Light-Cone M-theory. We can then look at every part of the phase diagram of this system from these three different viewpoints.

The scale of time-space non-commutativity of the NCOS theory is set by l_e. We fix M and $\sigma = \Sigma/l_e$, and vary the coupling $g = G_o\sqrt{M}$ and the temperature $t = Tl_e$. These are our four independent parameters. In figure 1, we plot $\ln t/\ln M$ on the vertical axis, and $\ln g/\ln M$ on the horizontal. Varying g corresponds to changing N with fixed M; this means that g is restricted to the range

$$0 \leq g \leq M \tag{1.14}$$

for $N > 1$. Hence, the diagram is truncated on the right. The set of available couplings is discrete. We will think of M as being a large integer and think of the set as being dense in the region of interest. This setup is most convenient from the perspective of the NCOS theory. In the S-dual SYM theory, it corresponds to exploring a space of U(N) SYM theories with different ranks for the gauge group; every vertical line on the diagram corresponds to a different SYM theory as a function of energy scale. We also choose the cycle size σ in the range $1 \leq \sigma \leq M$, keeping track of it through the variable $z \equiv \ln\sigma/\ln M$, with $0 \leq z \leq 1$.

The global structure of the diagram looks familiar from similar analysis applied to SYM theories without electric flux [16]. However, several new twists arise, and differences emerge that explore naturally the peculiarities of time-space non-commutativity. Before dwelling into details, let us make some general comments regarding these differences. Phases with different equations of state appear delineated in figure 1 by solid lines. Dotted lines correspond to duality transformations that patch together different supergravity vacua within a given phase. Shaded regions cannot be described by dual supergravity solutions. Single lines correspond to Gregory-LaFlamme transitions [18, 19], while double lines are associated with string

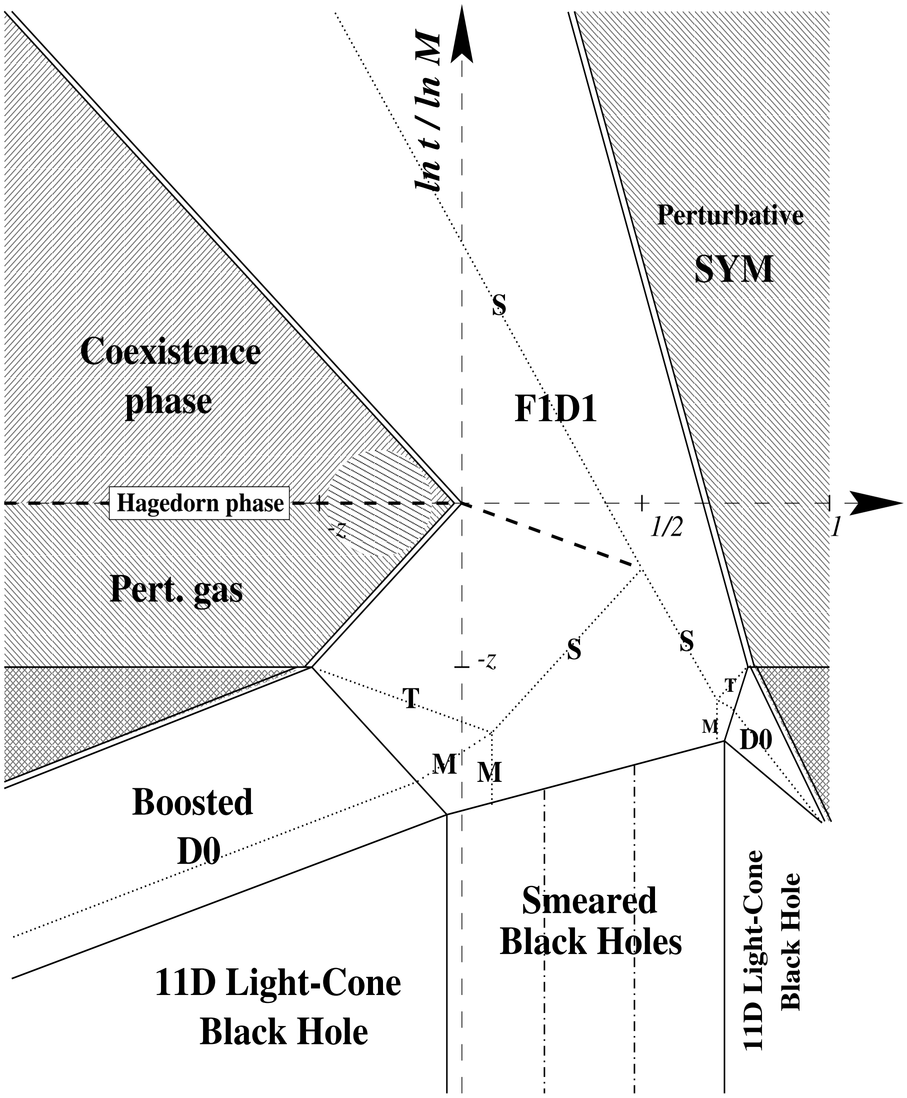

Figure 1: The phase diagram for the $(1+1)$-dimensional NCOS theory. We have defined $z \equiv \ln\sigma / \ln M$. Solid lines are Gregory-LaFlamme phase transitions; double lines are string-scale curvature regimes; dotted lines are duality transformations; shaded regions have no valid supergravity duals. The horizontal axis is the coupling, $\ln g / \ln M$.

scale curvatures in the corresponding geometries. On the right, and at high temperatures, the structure of the diagram is similar to the one arising in the case of zero electric flux. On the left, at temperatures around the NCOS string scale, NCOS stringy dynamics sets in. In the middle, at strong NCOS coupling g, the scale of non-commutativity gets dressed by a power of the coupling and the phase structure "folds" about a new energy scale

$$t \sim g^{-1/3} \equiv t_c\,. \tag{1.15}$$

The scale of non-commutativity as a function of g is shown in the figure by the dashed line. A similar phenomena was noted in [13] in the context of D3-branes. For the most part, well below and above the dashed line, the system may be described with commutative, field theoretical dynamics, albeit a complicated one. We next describe the dynamical details underlying the various phases, moving from the left towards the right.

The $(1+1)$-dimensional NCOS theory on the circle consists of three sectors: decoupled massless modes, corresponding to the U(1) modes of the S-dual SYM theory; massive open strings; and a sector of closed strings wrapping the compact cycle. The massless modes are irrelevant to the thermodynamics, as they describe the dynamics of the center of mass of the system. At $t \sim 1$, near the NCOS string scale, in the middle and left of the phase diagram, we have a few long open strings at high oscillator level, at the Hagedorn transition point. As we lower the temperature at weak coupling, it becomes thermodynamically more favorable to distribute the energy amongst many open strings, each at low oscillator number. In the presence of non-commutativity, as a result of (1.4), these low lying modes have longitudinal extent Δy proportional to the temperature [20, 21]

$$\Delta y \sim t\, l_e\,. \tag{1.16}$$

This is termed the non-commutative UV-IR relation. For $t \ll 1$, this Δy is sub-stringy. On the other hand, the Compton wavelength of these constituents grows with lower temperatures as [21]

$$\Delta y \sim \frac{l_e}{t}\,. \tag{1.17}$$

Therefore, the latter is the relation that sets their characteristic size for $t \ll 1$. At $t \sim 1/\sigma$, we then expect finite size effects to set in. Between $1/\sigma \ll t \ll 1$, we describe the phase by a gas of weakly interacting massless particles. Given M D-strings, there are M^2 species of these animals. The equation of state scales at leading order in the coupling as

$$E l_e \sim M^2 \sigma t^2\,. \tag{1.18}$$

As we move towards the right, there is a phase described by the near horizon geometry of the bound state of N fundamental strings and M D-strings, labeled in figure 1 as F1D1. Its equation of state is given by

$$E l_e \sim \sigma M^2 g^{-1} t^3\,, \tag{1.19}$$

reflecting strong coupling dynamics in the NCOS theory. The curvature of this geometry as measured at the horizon becomes string scale at

$$t \sim g\,. \tag{1.20}$$

Equating (1.18) with (1.19), we find (1.20), confirming the scenario just depicted.

The transition point $t \sim 1/\sigma$ sews onto a Gregory-LaFlamme transition at strong coupling to the right, further supporting the proposal to describe the phase above with a weakly interacting gas of point-like particles. At temperatures below $t \sim 1/\sigma$, the phase diagram mirrors structurally the right side. The region just below, shaded in dark in figure 1, is a transition phase whose dynamics has been a mystery even in the zero electric flux case [16]. Further below, we reach a phase that can be described by a supergravity vacuum, that of boosted D0-branes localized along the compact cycle. The transition point is where the curvature of the geometry as measured at the horizon becomes string scale. For temperatures even lower, we connect to phases in Light-Cone M-theory, the Matrix theory realization of this setup. We will come to this regime later.

Coming back to the $t \sim 1$ point, we increase the temperature, at fixed coupling, away from the Hagedorn phase where the thermodynamics is described by a few long NCOS open strings. Evidence was presented in [15] that the sector of wrapped closed strings of the NCOS theory may play an important role in trusting the system past its Hagedorn "limiting" temperature. Indeed, we can easily see this phenomenon here: the equation of state of such closed strings can be read off (1.13), and has the form

$$El_e \sim \omega\sigma t^2 \,, \tag{1.21}$$

where ω is the number of windings on the circle. The maximum value for ω is N, the number of D-strings in the S-dual picture. Therefore, as we increase the temperature from $t \sim 1$, the system may distribute its free energy amongst excited closed strings that split off the NCOS open strings, gradually reaching the energy (1.21) with $\omega = N$. In fact, from the side of the supergravity dual, we find that the curvature scale at the horizon at these high temperatures becomes string scale around

$$t \sim \frac{1}{g} \,. \tag{1.22}$$

This is precisely the temperature scale at which eq. (1.21), with $\omega = N$, equals the equation of state at strong coupling (1.19), confirming the scenario described. The phase between $1 \ll t \ll 1/g$ is then described by a coexistence phase of open and closed strings. The existence of such a coexistence region also arose in the context of the commutative SYM theory where the analogous phase is that of the Matrix string [22, 16]; there, it was proposed that such a transition region is needed to "unwind" the Z_N holonomy from the Matrix string configuration [23, 24]. In the current scenario, the presence of D0-brane charge in the Light-Cone IIA theory in question may change the dynamics. But at these high temperatures, eq. (1.21) with $\omega = N$ is the equation of state of a IIA Matrix string with N units of Light-Cone momentum

$$E \sim \frac{\alpha'}{N\Sigma} \frac{S^2}{\alpha'} \,. \tag{1.23}$$

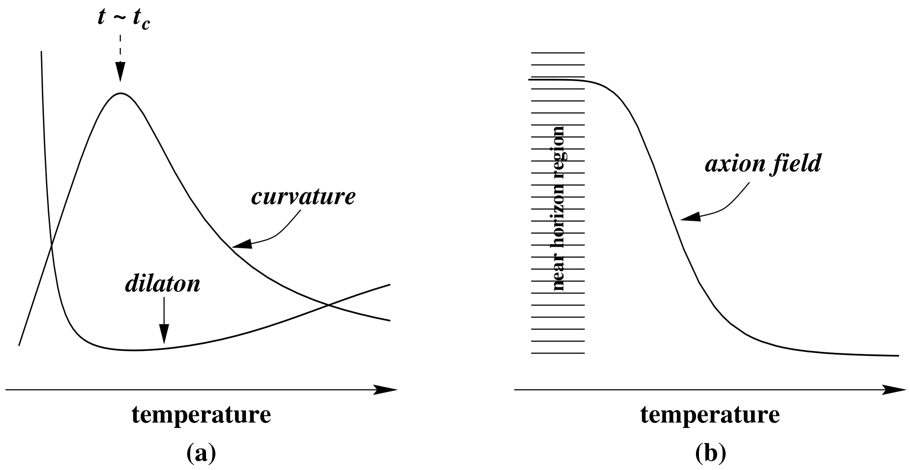

Figure 2: (a) The curvature scale and coupling in the (N,M) geometry in the near horizon region. (b) The axion field in the same geometry; the attractor value is N/M.

However, as we shall see below, this phase sews onto very different physics at lower temperatures.

In passing, let us note that both transition curves appearing in eqs. (1.20) and (1.22) relate the temperature to the open string coupling G_o in the combination $g = G_o\sqrt{M}$; M does not appear in these equations independently. This is a motivation for identifying the relevant coupling of the perturbative expansion of the theory as g and not G_o.

We now move to the middle section of the phase diagram, where the dynamics can be described by supergravity duals. The dominant patch in the middle phase consists of the near horizon geometry of the (N,M) string. The feature of interest here is the emergence of a new energy scale for non-commutativity given by eq. (1.15). The curvature at the horizon and the string coupling are plotted in figure $2a$ Both exhibit an extremum at the scale t_c. At weaker couplings towards the left of the diagram, the effect of the flip in the curvature scale was elaborated on above; it corresponds to the transition of dynamics between the NCOS open and closed string sectors. This aspect does not effect the Holographic UV-IR relation: we may have expected that the dispersion relation in this background geometry is such that perturbations seek areas of smaller curvature for lower energies; and this is indeed the case. However, there also exists a compensating red shift factor dressing the time at infinity, and hence energy appears to still flow to lower scales for lower values of the radial coordinate of the geometry.

The flip in the dilaton flow at fixed NCOS coupling must reflect the effects of the spreading of the degrees of freedom in the longitudinal direction as a function of energy. Earlier, we argued qualitatively that we may expect that the constituents of the system will spread in size as we move to lower and higher energies from the scale of non-commutativity because of the quantum mechanical and stringy uncertainty relations, respectively (see eqs. (1.17) and (1.16)). Overlapping wavefunctions would then imply stronger effective coupling between the degrees of freedom for energy scales away from t_c. This then explains the peculiar behavior of the dilaton field.[1] A phenomena of identical dynamics arose in the context of D3-branes and T-duality in [25]. For high temperatures, we sew onto the S-dual geometry of (-M,N) strings. For lower temperatures, we need to make use of another element of the $SL(2, \mathbb{Z})$ duality group. A similar complication was also present in [25] in the context of Morita equivalence. The problem arises here since, for $t \ll t_c$, the axion field that vanishes at infinity is attracted to a fixed value in the near horizon region, complicating the action of the S-duality group. This value of the axion field is a rational N/M; on the other side of the duality, it is the inverse. Figure 2b shows the behavior of the field in this vacuum. Note also that the geometry for $t \ll t_c$ is identical (up to a coordinate transformation) to the one appearing on the right of the diagram in the S-dual SYM field theory frame, further supporting the notion that at low temperatures the non commutative (hence stringy) aspects of the NCOS dynamics become unimportant.

As we move to lower temperatures in the middle of figure 1, a T-duality on Σ is required leading us to a phase of smeared boosted D0-branes; and then a Gregory-LaFlamme transition on Σ settles the system into M localized D0-branes with N units of boost. Further down, we lift to M-theory, first to an oblique wave on the torus, then to a boosted black hole. For these low energies, we connect to a generic finite temperature vacuum in Light-Cone M-theory, suggesting that the NCOS theory can describe Light-Cone M-theory with an additional boost [5]. We will elaborate on this issue further down.

To the right of the diagram, at high temperatures, the strong coupling behavior of the NCOS theory can be described by weakly coupled $(1+1)$-dimensional SYM degrees of freedom. The Yang-Mills coupling is given by

$$g_Y{}^2 = \frac{M^2}{g^4 \alpha_e} = \left(\frac{N}{M}\right)^2 \alpha_e{}^{-1}, \tag{1.24}$$

so that the coupling becomes a rational number when measured at the NCOS string scale. The phase structure in the right half of the diagram is virtually identical to the one that arises in the $(1+1)$-dimensional SYM case with zero electric flux. We will therefore only briefly discuss this part and refer the reader to [16]. We note that, in the current scenario, each different vertical line on our diagram corresponds to a different $U(N)$ SYM theory, since N is varying with M being fixed. But each vertical line maps onto a SYM theory with a shifted Yang-Mills coupling, such that the effective Yang-Mills coupling measured at a fixed temperature is increasing when one moves toward the left. This coupling $g_{\rm eff}^2 = g_Y{}^2 N T^{-2}$, T being the temperature, becomes of order one when measured at the double line between the perturbative

[1] We are assuming here that the flow of the dilaton is correlated with the flow of the effective coupling of the Holographic dual degrees of freedom.

SYM gas and the F1D1 phases.[2] The equation of state of the perturbative phase is given by (1.21) with $\omega = N^2$. More generally, the M units of electric flux in the SYM theory appear not to effect equations of state or critical phenomena arising in this part of the phase diagram. The region that appears checkered between the SYM gas and D0 phases involves a transition phenomenon that organizes the SYM excitations into its zero modes at fixed entropy and large coupling. It appears in [16] as a single horizontal solid line. At lower temperatures, we dwell into phases of Light-Cone M-theory, the Matrix theory regime.

Let us next focus on the phases appearing at the lowest temperatures. As is typical in these systems, the preferred configurations are eleven-dimensional black holes in Light-Cone M-theory. On the left and right sides, these black holes are localized and carry oblique momentum on a two torus. Both momenta survive the decoupling limit; but the dominant one sets the Light-Cone direction. The Planck scale on both sides is given in terms of the NCOS parameters by

$$l_{\rm pl}^3 = \frac{\alpha'^3}{l_e{}^3}\frac{M^2}{\sigma g^4} \longrightarrow 0\,. \tag{1.25}$$

On the left side, the eleventh cycle and the cycle R related to Σ scale as

$$\frac{l_{\rm pl}^2}{R} = \sigma^{1/3} M^{-2/3} g^{4/3} l_e\,, \qquad \frac{l_{\rm pl}}{R_{11}} = \sigma^{2/3} M^{2/3} g^{-4/3}\,, \tag{1.26}$$

which are held fixed in the decoupling limit. We see that the Light-Cone circle is R, not R_{11}. On the right side, these relations are the same with $R \leftrightarrow R_{11}$, reflecting an action of the modular group of the torus in between. Hence, on the right side, the Light-Cone direction is the eleventh direction. Correspondingly, for both sides, the momentum in the Light-Cone direction is N units, the dominant momentum charge in this decoupling limit. The equation of state of the phase on the left is

$$E l_e \sim g^{-6/7} \sigma^{3/7} M^{12/7} t^{16/7} \sim \frac{R}{N}\frac{S^{16/9}}{l_{\rm pl}^2}\,; \tag{1.27}$$

and the same on the right with $R \to R_{11}$. The characteristic Light-Cone scaling in this equation indicates that indeed the dominant momentum is set by the charge N on both sides of the diagram.

In the middle and at low temperatures, we have phases of smeared boosted eleven-dimensional black holes. The corresponding Gregory-LaFlamme transitions are complicated by the fact that the momentum is oblique on the two torus. We

[2] To structurally relate to [16, figure 1], we can identify the entropy there with our vertical axis here, while the horizontal axis there is the Yang-Mills coupling measured at the IR cutoff Σ; in our case, this corresponds to the combination $M^4\sigma^2/g^6$. The diagram in [16] does not truncate on the right since one varies the relative scales between the Yang-Mills coupling and the box size Σ, i.e. σ in the current language, while keep the rank of the gauge group N fixed.

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qualitatively expect three intermediate phases; a middle one smeared on two cycles, and two adjacent ones on either sides smeared on a single cycle. There may exist different scaling regimes for these Gregory-LaFlamme transitions depending on the size of Σ. In addition, the torus is skewed in the near horizon region due to the presence of a non-zero axion field in the IIB theory. A proper analysis requires a better understanding of the dynamics of a black hole with an oblique boost on a skewed two torus. We contend ourselves here with the qualitative picture just presented. For higher temperatures, and in the middle of the diagram, these complications appear in the form of duality transformations in IIB theory. The modular group of the torus becomes the $\mathrm{SL}(2,\mathbb{Z})$ S-duality group; the gap between the (N,M) and $(-M,N)$ supergravity vacua is to be filled with a finite series of such transformations. It appears from the global structure that we are guaranteed to find the proper supergravity framework in every region of this phase. However, one may need to numerically study each different combination (N,M) as in [25]. None of these issues will affect the thermodynamics of the middle phase and its equation of state. We will therefore not dwell in the details of these duality transformations.

The final conclusion we are lead to from the lower part of the diagram is that $(1+1)$-dimensional NCOS theory can describe Light-Cone M-theory with an additional boost. The map between the parameters of the two theories is given in eqs. (1.25) and (1.26).

Throughout the discussion, the parameter space explored was restricted to $0 \leq z \leq 1$ and $M \gg 1$. Let us next comment one some of the limiting regimes of these variables. First, we note that the coupling g lives in a discrete set. For M large, this set is reasonably dense; for example, for M about a hundred, $\ln g/\ln M$ changes only by a few percents for each step near the right border of the diagram; and the distribution becomes denser towards the left. As M decreases, the right half of the diagram is progressively less explored by the available values of the discrete coupling. Drawing the liming case $M=1$ as a separate diagram illustrates best this trend. Figure 3 shows the phase diagram for $M=1$; the axis are $\ln t$ versus $\ln g$.

Let us next vary the size of the circle z. For Σ in the range $l_e \leq \Sigma \leq l_e M$, the structure of the diagram is given by figure 1. For substringy values of Σ, the analysis on the left side of the diagram must be revised. We encounter a problem at the Gregory-LaFlamme critical point, where the transition curve becomes inconsistent with the equation of state on the other side of the phase. Further to the left, we expect that the size of the effective degrees of freedom is superstringy; but yet the cycle Σ is substringy. Given these two problems, it then appears conceptually problematic to extend the thermodynamics to substringy cycle sizes. Perhaps this indicates that, due to the non-commutativity of the time-space coordinates, it is inconsistent or, better said, dynamically disfavored to compactify the system on a circle smaller than the length scale set by the non-commutativity. For larger values of Σ, the point $\Sigma \sim M l_e$ is special, as can be seen from the labels in figure 4. Roughly speaking,

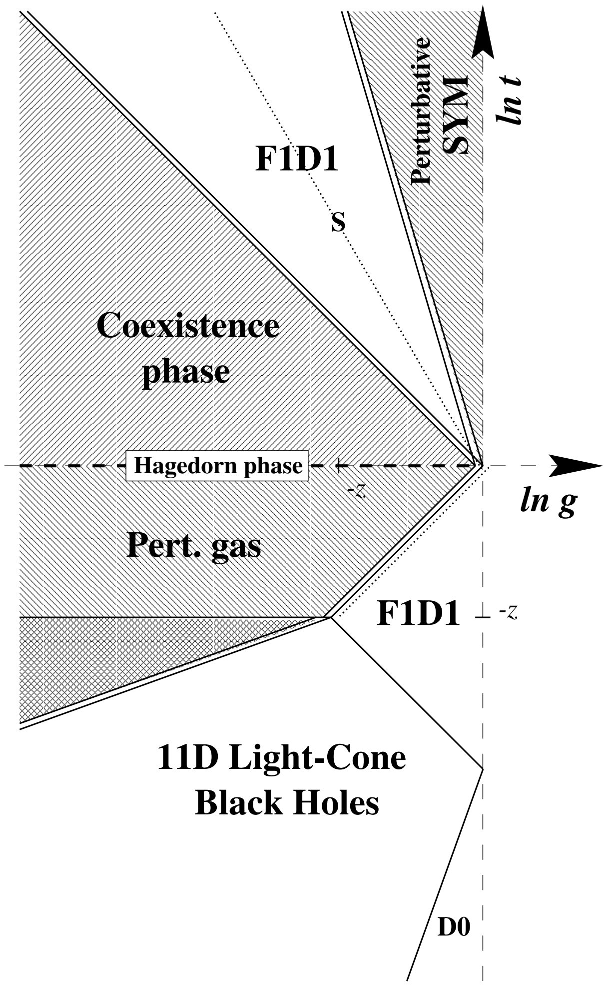

Figure 3: The phase diagram of (1 + 1)-dimensional NCOS theory with one unit of D string charge.

the low temperature part and the right sides of figure 1 are scaled out of focus. A point of contention is then the Hagedorn transition, since we have argued above that it corresponds to the production of closed strings winding the cycle. We then should expect it to be scaled out in the large cycle size limit. The phenomenon is also coupling sensitive as it involves a process where open strings join their endpoints to form wrapped closed strings. A resolution of this puzzle is possibly in the following observation: the region just to the right of the Hagedorn crossing point, the wedge between the supergravity descriptions, cannot be explained through the simplistic analysis we have presented. This is because the coupling there is becoming big, while the shape of the wedge we have drawn is an approximate one that breaks down precisely in this part of the diagram, even within our supergravity analysis. We then may expect that the microscopic dynamics in this area to the left of $g \sim 1/\sigma$, shaded differently in figure 1, involves dynamics that needs to be understood by a microscopic calculation. Hence, for $z \to \infty$, we propose that the Hagedorn transition we described above is scaled away towards the left, and the new unknown dynamics takes over; the latter should obviously not involve the NCOS closed string sector. The system exhibits a remarkable level of richness and more analysis is needed to decode all of its structures.

1.3 Discussion

We studied a region of the thermodynamic parameter space of the (1+1)-dimensional NCOS theory on a circle. Our results can be summarized as follows:

- We showed evidence that the perturbative expansion of the NCOS theory is with respect to the parameter $g \equiv G_o\sqrt{M}$.
- We have found that, at strong coupling, the scale of non-commutativity gets dressed by a power of the NCOS string coupling. We get the new characteristic length scale $l_e g^{1/3}$.
- The full $\mathrm{SL}(2,\mathbb{Z})$ duality group is needed to explore the space of relevant supergravity vacua. In particular, we have the peculiar feature that the dilaton grows at high and low energies at fixed coupling, reflecting a UV-IR mixing in the non-commutative dynamics.
- Stringy non-commutativity is relevant to the thermodynamics at around the energy scale of non-commutativity, and above it at weak coupling; away from these regimes, the dynamics appears to simplify.
- The NCOS theory can describe Light-Cone M-theory with an additional boost. One of the two charges of the system singles out the Light-Cone direction on a two torus.
- The Hagedorn transition may indeed involve the transfer of energy from the open string sector to wrapped closed strings, as suggested in [15]. This is correlated with a peculiar behavior in the curvature scale of the dual geometry.
- We find a possible lower bound on the size of the circle of compactification; mainly, the scale set by the non-commutativity. We also note an interesting special point at $\Sigma \sim M l_e$.

The system we have studied appears to be very rich in structure. We have presented a first analysis and a great deal remains to be explored. The phases appearing on the left of figure 1 need to be studied in detail from a microscopic viewpoint. Our picture of the phase structure in this region may get refined through a better understanding of the underlying stringy dynamics. It would be interesting to see whether the small cycle size limit can yield new insight into the theory. Such an analysis will also lead to a better understanding of this phase space from the point of view of the SYM theory. The flipping of the dilaton field in the (N,M) geometry may be indicative of interesting behavior in the beta function of the SYM theory. A renormalization group analysis at strong coupling is possible and may lead interesting results about the dynamics of the gauge theory in the presence of electric flux [26]. It may also be useful to look at the higher-dimensional cases and OM theory using the same technology. Finally, understanding the dynamics of the Gregory-LaFlamme transitions on a skewed two torus may be an amusing problem from the point of view of gravitational dynamics.

In perspective, given the control one has on the perturbative and strong coupling aspects of this particular stringy system, it appears to be a prime setting to explore stringy non-commutative physics decoupled from complications of a gravitational nature.

2. The details

In this section, we sketch the technical details involved in constructing the phase diagram of figure 1. There are twelve distinct phases in the system, eight of which can be described through their supergravity duals. We have labeled the various transition curves with small latin letters, bulk phases with capital letters, and patches of geometries by a letter and a number. The reader is referred to figure 4 for a roadmap. Our starting point is the central phase in figure 4.

The D1F1 phase (A). The IIB theory is parameterized by the string coupling $g_{\rm str}$ and the string scale α'. The metric of interest is the one found in [27] for the bound state of N fundamental strings and M D strings. In the string frame, it looks like

$$ds^2_{\rm str} = g_{\rm str}\left(\frac{K}{L}\right)^{1/2}\left[A^{-1/2}\left(-f\,dt^2+\Sigma^2dy^2\right)+A^{1/2}\left(f^{-1}dr^2+r^2d\Omega_7^2\right)\right]. \qquad (2.1)$$

The coordinate y is compact with size 2π. The NSNS and RR gauge fields are

$$B_{ty} = g^2_{\rm str}\Sigma\frac{N}{M}A^{-1}L^{-1/2}\,, \qquad A_{ty} = \Sigma A^{-1}L^{-1/2}\,; \qquad (2.2)$$

and the dilaton and axion get turned on as well

$$e^{\phi} = g_{\rm str}A^{1/2}\frac{K}{L}\,, \qquad \chi = \frac{N}{M}A^{-1}\frac{A-1}{K}\,. \qquad (2.3)$$

The harmonic functions are

$$A = 1+\frac{q^6}{r^6}\,, \qquad f = 1-\frac{r_0^6}{r^6}\,, \qquad (2.4)$$

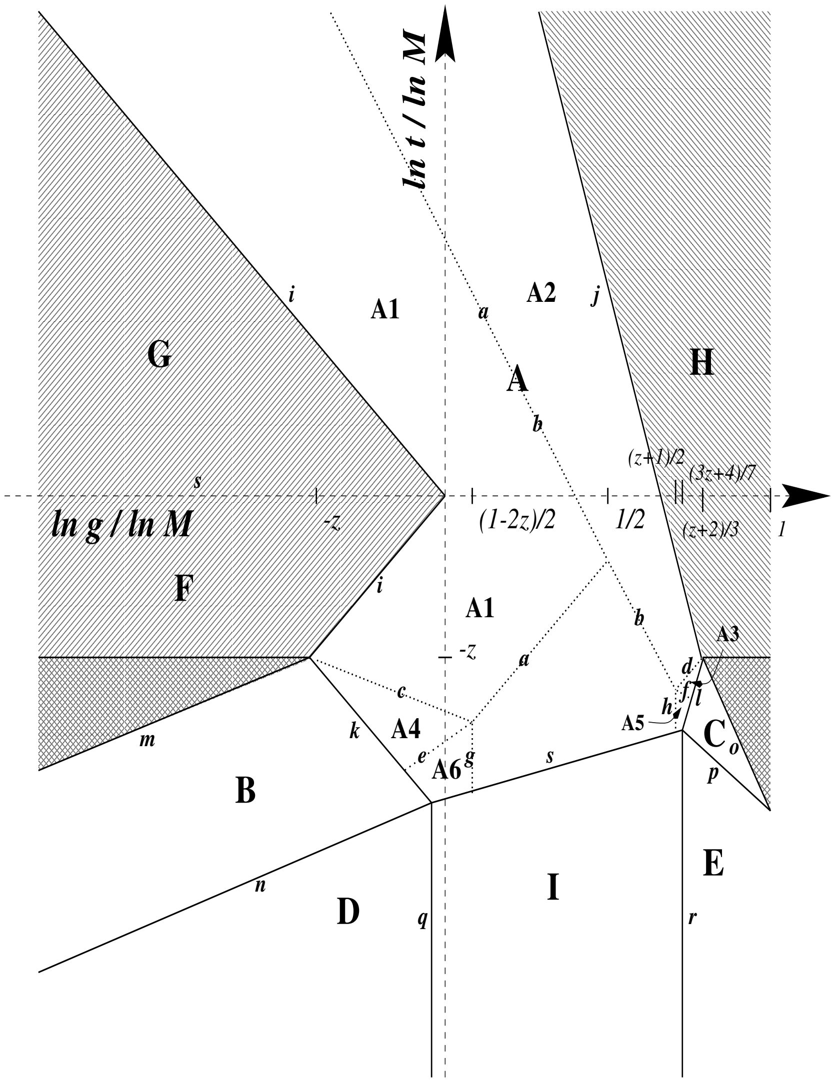

Figure 4: The NCOS phase diagram: roadmap used in section 2.

with

$$q^6 \equiv \frac{32\pi^2 M}{g_{\rm str}^2} \alpha'^3 L^{1/2} \,, \tag{2.5}$$

and r_0 the location of the thermodynamic horizon. The other functions appearing in (2.1) are

$$K \equiv 1 + A^{-1} \left(\frac{N g_{\rm str}}{M} \right)^2 , \qquad L \equiv 1 + \left(\frac{N g_{\rm str}}{M} \right)^2 . \tag{2.6}$$

Note that we have chosen coordinates such that the S-dual metric asymptotes to Minkowski space. This corresponds to the choice advertised in (1.1). And the B_{ty} field attains the value (1.2) at infinity. The coordinate y is compactified on a circle of radius Σ.

The decoupling limit is obtained by

$$\alpha' \longrightarrow 0\,, \qquad g_{\rm str}\alpha' = \alpha_e G_s^2 \text{ held fixed}\,, \tag{2.7}$$

while $G_s \to M/N$. We also need to keep $U \equiv r/\alpha'$ fixed.[3] In this limit, the metric (2.1) becomes (A1)

$$ds^2_{\rm str} = \alpha' G_s \Delta^{1/2}\left[\frac{l_e G_s}{\sqrt{32\pi^2 N}} U^3\left(-f dt^2 + \Sigma^2 dy^2\right) + \right. \\ \left. + \frac{\sqrt{32\pi^2 N}}{l_e G_s} U^{-3}\left(f^{-1} dU^2 + U^2 d\Omega_7^2\right)\right]. \tag{2.8}$$

The other fields become

$$B_{ty} = \alpha' \Sigma \frac{{\alpha_e}^2 G_s^4 U^6}{32\pi^2 N}\,, \qquad A_{ty} = {\alpha'}^3 \Sigma \frac{G_s U^6}{32\pi^2 N}\,; \tag{2.9}$$

$$e^\phi = \frac{\sqrt{32\pi^2 N}}{{l_e}^3 G_s U^3}\Delta\,, \qquad \chi = G_s^{-1}\Delta^{-1}\,. \tag{2.10}$$

The dispersion relation of perturbations propagating in this curved space dictates a relation between the radial extent U and the energy scale of the perturbation. The holographic UV-IR relation yields

$$U_0^2 \sim \frac{T N^{1/2}}{l_e G_s} \sim \frac{M^2}{g^3}\frac{t}{\alpha_e}\,, \tag{2.11}$$

where T is the corresponding energy scale or, in our case, the Hawking temperature for U_0 the location of the horizon. The temperature in NCOS string scale units is denoted by $t \equiv T l_e$, and Δ is defined by

$$\Delta \equiv 1 + \frac{{\alpha_e}^3 G_s^4}{32\pi^2 N} U^6 = 1 + \frac{g}{32\pi^2} t^3\,, \tag{2.12}$$

with $g \equiv G_s\sqrt{N} = G_0\sqrt{M}$ as defined in the Introduction. $G_0 = \sqrt{G_s}$ is the string coupling of the NCOS theory. There are two temperature regimes in this system, separated by the scale $t \sim g^{-1/3} \equiv t_c$. For low temperatures, one can replace $\Delta \to 1$ in the equations above. While the decoupling limit is a strict scaling limit, this is an approximation to a certain regime lying within the energy window that is survived by the limit. We also define the dimensionless parameter $\sigma \equiv \Sigma/l_e$.

The duality boundaries of this vacuum are as follows:

- The dilaton at the horizon $U = U_0$ becomes big unless (curve (a))

$$t \gg M^{-2/3} g \Delta^{2/3}\,. \tag{2.13}$$

[3] Note that in the more conventional coordinates $r \to g_{\rm str}^{1/2} r$, the radial coordinates would scale as $\sqrt{\alpha'}$. Our choice corresponds to a canonical normalization in the S-dual SYM frame instead.

For high temperatures $t \gg t_c$, this corresponds to

$$t \ll M^{2/3} g^{-5/3} , \tag{2.14}$$

while for lower temperatures,

$$t \gg M^{-2/3} g . \tag{2.15}$$

Beyond these points, an element of the SL(2, Z) S-duality group needs to be applied.

- The curvature scale of the metric as measured at its horizon becomes string scale unless (curve (i))

$$t \ll g\Delta . \tag{2.16}$$

The high temperature regime yields

$$t \gg g^{-1} , \tag{2.17}$$

while the low temperature scenario is

$$t \ll g . \tag{2.18}$$

Beyond these point, the perturbative 1+1 NCOS theory emerges with coupling constant given by $g = G_o\sqrt{M}$, string scale α_e, scale of non-commutativity set by α_e, and M Chan-Paton indices on the string endpoints.

- The size of the circle on which the bound state is wrapped becomes string scale, as measured at the horizon, unless (curve (c))

$$t \gg \sigma^{-4/3} g^{-1/3} \Delta^{-1/3} \Longrightarrow t \gg g^{-1/3} \sigma^{-4/3} . \tag{2.19}$$

In this case, there is a single temperature regime if $\sigma > 1$. Otherwise, we get $t \gg \sigma^{-2/3} g^{-1/3}$. Many of our subsequent equations will be sensitive to the assumption that the cycle size is superstringy or at around the NCOS string scale. We will not discuss the substringy regime in this work for the reasons discussed in the Introduction. Beyond the regime set by eq. (2.19), one needs to look at the T-dual geometry.

For high temperatures $t \gg t_c$, we sew onto the strong coupling phase by applying the S-duality transformation

$$\begin{pmatrix} -M \\ N \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} N \\ M \end{pmatrix} \tag{2.20}$$

on the metric (2.1) and (2.2). This is the vacuum adjacent to the patch described above, appearing to its right in figure 4. The metric becomes (A2)

$$ds^2_{\rm str} = \alpha' \left[\frac{l_e G_s}{\sqrt{32\pi^2 N}} U^3 \left(-f\,dt^2 + \Sigma^2 dy^2\right) + \right. \\ \left. + \frac{\sqrt{32\pi^2 N}}{l_e G_s} U^{-3} \left(f^{-1} dU^2 + U^2 d\Omega_7^2\right) \right], \tag{2.21}$$

with the NSNS and RR gauge fields exchanged

$$B'_{ty} = -A_{ty}\,, \qquad A'_{ty} = B_{ty}\,. \tag{2.22}$$

The dilaton and axion become

$$e^{\phi'} = \frac{\sqrt{32\pi^2 N}}{{l_e}^3 G_s^3 U^3}\,, \qquad \chi' = -G_s = -\frac{M}{N}\,. \tag{2.23}$$

Note that the axion field is attracted to a ratio of integers in the near horizon region. The metric, the dilaton and the RR gauge field are identical to the case dual to $1+1$ SYM without an electric field and an asymptotic constant RR gauge field. For $t \ll t_c$, we note that the previous metric, eq. (2.8), is of similar form; in that case, the axion field (eq. (2.10) with $\Delta \sim 1$), is inverted.

The boundaries of this vacuum are as follows:

- The dilaton is big at the horizon unless (b)

  $$t \gg M^{2/3} g^{-5/3}\,. \tag{2.24}$$

  Examining this statement in conjunction to (2.13), reveals that we have a gap between this vacuum configuration and the previous one for $t \ll t_c$. The resolution of this puzzle lies in the realization that, even though the axion field vanishes at infinity, it has a non-zero value in the near horizon region, so that the full $\mathrm{SL}(2,\mathbb{Z})$ S-duality group is to fill the gap with the appropriate vacua. The axion fields on both sides of this gap are ratios of M and N; one being the inverse of the other. We will come back to this issue at the end of this section.

- The curvature scale as measured at the horizon is too big in string units unless (j)

  $$t \ll M^2 g^{-3}\,. \tag{2.25}$$

  Beyond this point, the perturbative $(1+1)$-dimensional $\mathrm{U}(N)$ SYM theory emerges with M units of electric flux.

- The circle parameterized by y is too small in string units as measured at the horizon unless (d)

  $$t \gg \sigma^{-4/3} M^{-2/3} g\,. \tag{2.26}$$

  For smaller circles, we need to look at the T-dual configuration.

We next look at the T-dual of the $(-M,N)$ phase we just discussed. The IIA metric is given by (A3) (see for example [28])

$$ds^2_{\rm str} = -\alpha'\left(\frac{l_e G_s U^3}{\sqrt{32\pi^2 N}}\right) f\,dt^2 + \alpha'\frac{\sqrt{32\pi^2 N}}{l_e G_s \Sigma^2 U^3}dy^2 + \\ + \alpha'\frac{\sqrt{32\pi^2 N}}{l_e G_s U^3}\left(f^{-1}dU^2 + U^2 d\Omega_7^2\right), \tag{2.27}$$

with the dilaton

$$e^\phi = \frac{(32\pi^2 N)^{3/4}}{\Sigma {l_e}^{7/2} G_s^{7/2} U^{9/2}}. \tag{2.28}$$

There is one form flux due to the axion and the two form RR fields of the T-dual picture

$$A_t = \Sigma l_{\rm str}\frac{{\alpha_e}^2 G_s^4 U^6}{32\pi^2 N}, \qquad A_y = -l_{\rm str} G_s. \tag{2.29}$$

This configuration consists of boosted smeared D0-branes.

The new boundaries for this vacuum are:

- The string coupling is too big at the horizon unless (f)

  $$t \ll \sigma^{-4/9} M^{2/9} g^{-7/9}. \tag{2.30}$$

  We then lift to a smeared wave with oblique momentum on a two torus in an M-theory.

- The vacuum undergoes the Gregory-LaFlamme transition along y unless (l)

  $$t \ll \sigma^{-2} M^{-2} g^{-3}. \tag{2.31}$$

  The new phase beyond this point is described by localized boosted D0-branes.

Going to the opposite side of the phase diagram, we next look at the T-dual of the original (N,M) configuration. The IIA metric is (A4)

$$ds^2_{\rm str} = -\alpha'\frac{G_s^2 l_e U^3}{\Delta^{1/2}\sqrt{32\pi^2 N}}\left(\Delta f - \Delta + 1\right)dt^2 - 2\frac{\alpha'}{\Sigma}\frac{{l_e}^3 G_s^2 U^3}{\Delta^{1/2}\sqrt{32\pi^2 N}}dt\,dy + \\ + \frac{\alpha'}{\Sigma^2}\frac{\sqrt{32\pi^2 N}}{G_s^2 \Delta^{1/2} l_e U^3}dy^2 + \alpha'\Delta^{1/2}\frac{\sqrt{32\pi^2 N}}{l_e U^3}\left(f^{-1}dU^2 + U^2 d\Omega_7^2\right), \tag{2.32}$$

and describes boosted smeared D0-branes again. The coordinate y has period 2π. The dilaton is

$$e^\phi = \frac{(32\pi^2 N)^{3/4}\Delta^{3/4}}{{l_e}^{7/2} G_s^2 U^{9/2}\Sigma}, \tag{2.33}$$

while the one form gauge fields are given by

$$A_t = -l_{\rm str}\Sigma\Delta^{-1}\frac{{\alpha_e}^2 G_s^3 U^6}{32\pi^2 N}, \qquad A_y = l_{\rm str} G_s^{-1}\Delta^{-1}. \tag{2.34}$$

The new boundaries of this patch are:

- The Gregory-LaFlamme instability along y occurs unless (k)

$$t \gg \sigma^{-2} g^{-1}\,. \tag{2.35}$$

The new phase consists of boosted localized D0-branes.

- The dilaton is too big at the horizon unless (e)

$$t \gg \sigma^{-4/9} M^{-4/9} g^{5/9}\,. \tag{2.36}$$

Otherwise, we lift to M-theory and a configuration of oblique smeared waves on a torus.

We next look at the vacuum obtained by lifting the metric (2.27). The eleven-dimensional metric becomes (A5)

$$\begin{aligned} ds_{11}^2 = \alpha' {l_e}^{4/3} G_s^{4/3} \Sigma^{2/3} \left(f^{-1} dU^2 + U^2 d\Omega_7^2\right) + \\ + \alpha' \frac{{l_e}^{4/3} G_s^{4/3}}{\Sigma^{4/3}} dx^2 - \alpha' f \frac{{l_e}^{10/3} G_s^{10/3} \Sigma^{2/3} U^6}{32\pi^2 N} dt^2 + \\ + \frac{32\pi^2 \alpha'}{{l_e}^{14/3} G_s^{14/3} \Sigma^{4/3} N U^6} \left(N dx_{11} - M dy + \frac{{l_e}^4 G_s^4 \Sigma U^6}{32\pi^2} dt\right)^2 . \end{aligned} \tag{2.37}$$

x_{11} is compact with periodicity 2π, so that the combination $Nx_{11} - My$ respects the periodicity of the torus. Note also that we have chosen a somewhat unconventional normalization such that the M-theory energy scale is scaled by $g_{\rm str}^{1/3}$; i.e. we have lifted to M-theory using a dilaton field which asymptotes to $g_{\rm str}$. This is convenient in this setting since it makes the α' scaling of this region of space explicit.

There are two new boundaries to this phase:

- A reduction to IIA theory along x is needed unless (h)

$$g \gg M^{1/2} \sigma^{1/2}\,. \tag{2.38}$$

We come back to this issue at the end of the section.

- A localization transition occurs unless (s)

$$t \gg \sigma^{-1/2} M^{-1} g^{1/2}\,. \tag{2.39}$$

The new phase is a boosted black hole in M-theory. More about this phase below.

Jumping to the other side of the phase diagram, we next describe the M lift of the T dual of the (N, M) phase. The geometry is again that of smeared waves on a two torus in an M-theory (A6)

$$
\begin{aligned}
ds_{11}^2 = & \ \alpha' \frac{{l_e}^{4/3}}{\Sigma^{4/3} G_s^{2/3}} \left(dy - \frac{{l_e}^4 G_s^4 U^6 \Sigma}{32\pi^2 N} dt \right)^2 - \\
& - \alpha' \frac{{l_e}^{10/3} G_s^{10/3} \Sigma^{2/3} U^6}{32\pi^2 N} f dt^2 + \alpha' {l_e}^{4/3} G_s^{4/3} \Sigma^{2/3} \left(f^{-1} dU^2 + U^2 d\Omega_7^2 \right) + \\
& + \alpha' \frac{32\pi^2}{{l_e}^{14/3} G_s^{11/3} \Sigma^{4/3} M U^6} \left(M dx_{11} + N dy - {\alpha_e}^2 \frac{\Sigma G_s^4 U^6}{32\pi^2} dt \right)^2 ,
\end{aligned}
\tag{2.40}
$$

with the periodicities of x_{11} and y being 2π.

The boundaries are:

- We need to perform a reduction to IIA theory unless (g)

$$g \ll \sigma^{-1} M^{1/2} . \tag{2.41}$$

  The emerging vacuum is.

- And a localization transition occurs unless (s)

$$t \gg \sigma^{-1/2} M^{-1} g^{1/2} , \tag{2.42}$$

  as in (2.39). The emerging phase is a boosted black hole in M-theory.

We now have enough structure to describe all the boundaries of the D1-F1 phase. Strictly speaking however, to complete the discussion about all the supergravity vacua patching up this phase, we need to write down the metrics appearing in the gap region created in the middle of the phase. These various frames appearing in the middle patch are to be obtained by making use of the full SL$(2,\mathbb{Z})$ S-duality group. The fact that M and N are relatively primed is to play an important role in the existence of the appropriate duality transformation. We refrain from going into a detailed discussion since this will not affect any of the physical results of the work, mainly the phase transitions of the system.

The duality transformations sewing all these vacua into the D1F1 patch do not change the equation of state of this phase. The latter is given by

$$\varepsilon \equiv E l_e \sim \sigma M^2 g^{-1} t^3 . \tag{2.43}$$

This is identical to the equation of state obeyed by a system of M D1-branes. This point was emphasized in [14] in the context of the D3-brane system with magnetic and electric fluxes.

The boosted D0 phase (B). This phase arises from the metric (2.32) via a Gregory-LaFlamme transition along y. The equation of state is given by

$$\varepsilon \sim \sigma^{3/5} M^2 g^{-6/5} t^{14/5} \,. \tag{2.44}$$

To obtain this equation, we boost localized D0-branes

$$E^2 - p^2 = (M_0 + \mu)^2 \Longrightarrow E \simeq p + \frac{M_0^2}{2p} + \frac{M_0 \mu}{p} \,, \tag{2.45}$$

where M_0 is the BPS mass of M D0-branes, p is the boost momentum, and μ is the excitation energy above extremality. The limit follows in the scaling regime under consideration, i.e. the infinite boost scenario, since

$$M_0 = \frac{M\Sigma}{\alpha'} \longrightarrow \infty \,, \qquad p = \frac{N\Sigma}{\alpha'^2} g_{\rm str} \alpha' \longrightarrow \infty \,, \tag{2.46}$$

obtained from the BPS masses of M fundamental strings and N D-strings.[4] We then use the equation of state of D0-branes, being careful to substitute the appropriate asymptotic values of of the fields in the relevant frame

$$\mu \sim TS \sim \frac{(g_{\rm str}\alpha')^{1/6}}{l_{\rm str}\Sigma^{1/3}} S^{14/9} M^{-7/9} \times (g_{\rm str})^{1/2} \,. \tag{2.47}$$

The last factor of $g_{\rm str}^{1/2}$ arises from the choice of coordinates in the metric (2.1). Substituting these into (2.45) yields the equation of state (2.44). To check for consistency of our analysis, we equate eq. (2.44) with (2.43) to find the boundary transition curve

$$t \sim \sigma^{-2} g^{-1} \tag{2.48}$$

which is exactly (2.35), determined there independently from the shape of the geometry.

There are two new boundaries to this phase. Instead of looking for them in complicated geometries, to find these phase transition curves we will use a trick:

- From [16], we know that localized D0-branes at rest undergo two phase transitions; either when the entropy S is of M, the number of D0-branes (a Gregory-Laflamme transition to a boosted black hole); or when $S \sim M^2$, which is the point where the curvature scale at the horizon becomes string scale. These statements, making reference to the density of states and the number of D0-branes must be boost invariant. Using $E \sim TS$ in the equation of state (2.44), and $S \sim M^2$, we find the transition curve (m)

  $$t \sim \sigma^{-1/3} g^{2/3} \tag{2.49}$$

  in terms of the temperature of the system. Beyond this point, the perturbative NCOS may emerge.

[4] We remind the reader that the canonical normalization of the energy we use is in the $(-M, N)$ frame; i.e. the metric asymptotes to Minkowski in this frame.

- Similarly, a localization transition will occur at $S \sim M$, or (n)

$$t \sim \sigma^{-1/3} M^{-5/9} g^{2/3} \,. \tag{2.50}$$

Beyond this point, we have the phase of an eleven-dimensional localized black hole boosted obliquely along a torus.

The phase of D0-branes (C). The equation of state is given by

$$\varepsilon \sim g^{-2/5} M^{8/5} \sigma^{3/5} t^{14/5} \,. \tag{2.51}$$

Using the same trick as in the previous subsection, we get the transition curves:

- $S \sim N^2 = M^4/g^4$ for the string scale curvature point (o)

$$t \sim \sigma^{-1/3} M^{4/3} g^{-2} \,. \tag{2.52}$$

Beyond this temperature, we have a coexistence phase leading up to perturbative $1+1$ SYM gas.

- $S \sim N$ corresponds to the localization transition into a boosted black hole, which translates to the temperature (p)

$$t \sim \sigma^{-1/3} M^{2/9} g^{-8/9} \,. \tag{2.53}$$

The boosted 11D black holes (D and E). These are phases of localized black holes boosted obliquely on a two torus. The equation of state was given in (1.27); along with the parameters of the corresponding M-theories in eqs. (1.25) and (1.26). On curves of the qualitative form (q) and (r) in figure 4, these phases undergo Gregory-LaFlamme transitions to smeared and boosted black holes.

The smeared boosted black holes (I). The middle phases at low temperatures consist of various smeared and boosted black holes. The complications arising in this phase were discussed in the Introduction. We have not determined the exact scaling of the phase transitions curves separating these phases amongst themselves and from the rest of the phase diagram. There presence is required for the consistency of the structure of the diagram.

Perturbative 1+1 SYM and coexistence phase (H). The perturbative SYM phase obeys, to leading order in the Yang-Mills coupling, the equation of state

$$\varepsilon \sim \sigma N^2 t^2 \,. \tag{2.54}$$

Its effective coupling becomes of order one one both curves bounding it on the diagram. The lower one is also associated with finite size effects.

Perturbative 1+1 NCOS and coexistence phase (F and G). These phases can be understood through a thermodynamic interplay between the open and closed string sectors of the NCOS theory. The details were discussed in the Introduction.

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